

Gate teleportation using Parallel and Series entanglement distributions on arbitrary remote states

Debashis Saha^{}, Sanket Nandan[†] and Prasanta K. Panigrahi[‡]*
^{*,‡}*Indian Institute of Science Education and Research Kolkata,
 Mohanpur Campus, Nadia-741252, West Bengal, India*
[†]*Indian Institute of Science Education and Research Pune,
 Dr. Homi Bhabha Road, Pashan, Pune-411008, Maharashtra, India*

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Abstract

We consider two different entanglement distributions; Series and Parallel, between remote parties for implementing non-local controlled-Unitary gates. In parallel network, each of the control parties shares one entangled state with the target party and none of the control parties shares entanglement between them. In series network, the entanglement distribution forms a series, such that the control parties share entanglement with the adjacent control parties and only one of them shares entanglement with the target party, which is located at the end of the network. We demonstrate the protocol of simultaneous teleportation of controlled operation through both the distributions of Bell state entangled channels using local operations and classical communications (LOCC). For the parallel network, the communication cost of implementing controlled-Unitary gate on ‘n’ qubit arbitrary state is $(n-1)$ ebits and $2(n-1)$ cbits. Whereas, in general the series network allows controlled-Hermitian gate implementation on arbitrary state using $(n-1)$ ebits and $(n^2+n-2)/2$ cbits. We then explicate the protocol for complete teleportation of controlled-controlled-Unitary gate, a generalized form of Toffoli gate and subsequently generalize the protocol for ‘n’-qubit gate.

Keywords:

Bell state, controlled-Unitary gate, controlled-Hermitian gate, Toffoli gate, Gate teleportation.

1 INTRODUCTION

The counter-intuitive and one of the most striking features of the quantum world is entanglement, which has found practical use in the field of communication and cryptography [1]. Various quantum communication protocols, like qubit teleportation [2,3], superdense coding [4], quantum information splitting [5], secret sharing [6], remote state preparation [7,8] have been theoretically and experimentally demonstrated through entangled channels. Gate teleportation is another important protocol implementing a multi-partite quantum gates between qubits, which are spatially distributed. The protocol requires application of a unitary gate, that can not be decomposed into individual local operations i.e., $|\psi\rangle_{AB} \rightarrow \mathcal{U}|\psi\rangle_{AB}$, where $\mathcal{U} \neq \mathcal{U}_A \otimes \mathcal{U}_B$. This can be achieved using entangled channels shared by the remote parties. In the context of distributed quantum computing, it is necessary to produce controlled operations between remote parties, which may be more than two in number. Together with the local single particle operations, these can produce the necessary entanglement between remote parties for implementation of quantum tasks.

As is well known, controlled-NOT, together with the single qubit Hadamard gate, form the universal gates to which other gates can be decomposed [9]. Controlled-Unitary gates play significant role in quantum computation. In principle, these gates can be teleported, using only CNOT gate teleportation protocol. Involving less entanglement and communication costs, several protocols have been carried out implementing non-local multi-partite operations

^{*}debashis169@gmail.com

[†]sanketnandan@gmail.com

[‡]pprasanta@iiserkol.ac.in

locally by LOCC using entangled channels[10-20] and qubit communication [21,22]. Probabilistic and deterministic gate teleportation using non-maximally entangled state has been exploited [23-26]. Assisted with linear optical manipulations, photon entanglement produced from parametric down-conversion, and post-selection from the coincidence measurements, Huang *et al.* [27] teleported the CNOT gate experimentally. In contrast with the other protocols, we consider arbitrary multipartite state, either product or entangled, where all the qubits are remote placed and explicate the protocol of simultaneous and ‘n’ qubit controlled operation in two different entanglement distribution.

The paper is organized as follows. We start with a three party scenario, where Alice and Bob simultaneously teleport controlled-Unitary gate to Charlie in parallel entangled network, which is then generalized for arbitrary multi-partite state. Sect. II deals the series entanglement distribution of Bell states among three distant parties to simultaneously teleport controlled-Hermitian gates on ‘n’ remote parties. We then proceed to teleport controlled-controlled-Unitary gate and generalize it to n -controlled (n parties having the controls) Unitary gate teleportation. Sect. IV is devoted to conclusion and directions for future work.

2 SIMULTANEOUS TELEPORTATION OF CONTROLLED-UNITARY GATE USING PARALLEL NETWORK

In this configuration, we start with a system of three parties (fig. 1), where Alice and Bob are two control parties and Charlie is the target party. The three qubit arbitrary state is possessed by the parties having one qubit each. Alice and Charlie share one EPR pair between A and C_1 :

$$|\Phi\rangle_{AC_1} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (1)$$

Bob and Charlie share another EPR pair between B and C_2 :

$$|\Phi\rangle_{BC_2} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (2)$$

Thus there is no connection between the control parties and each of the control parties shares entanglement with the target party forming a parallel entanglement connection. The unknown input state comprising of the qubits a, b and c can be expressed as,

$$|\psi\rangle_{abc} = (d_0|000\rangle + d_1|001\rangle + d_2|010\rangle + d_3|011\rangle + d_4|100\rangle + d_5|101\rangle + d_6|110\rangle + d_7|111\rangle), \quad (3)$$

with $\sum_{i=0}^7 |d_i|^2 = 1$. The combined state of all the qubits possessed by Alice, Bob and Charlie is given by,

$$\begin{aligned} |\zeta\rangle_{aAbBC_1C_2c} &= (|\psi\rangle_{abc} \otimes |\Phi\rangle_{AC_1} \otimes |\Phi\rangle_{BC_2}) \\ &= \frac{1}{2} [(d_0|0000000\rangle + d_1|0000001\rangle + d_2|0010000\rangle + d_3|0010001\rangle \\ &\quad + d_4|1000000\rangle + d_5|1000001\rangle + d_6|1010000\rangle + d_7|1010001\rangle) \\ &\quad + (d_0|0001010\rangle + d_1|0001011\rangle + d_2|0011010\rangle + d_3|0011011\rangle \\ &\quad + d_4|1001010\rangle + d_5|1001011\rangle + d_6|1011010\rangle + d_7|1011011\rangle) \\ &\quad + (d_0|0100100\rangle + d_1|0100101\rangle + d_2|0110100\rangle + d_3|0110101\rangle \\ &\quad + d_4|1100100\rangle + d_5|1100101\rangle + d_6|1110100\rangle + d_7|1110101\rangle) \\ &\quad + (d_0|0101110\rangle + d_1|0101111\rangle + d_2|0111110\rangle + d_3|0111111\rangle \\ &\quad + d_4|1101110\rangle + d_5|1101111\rangle + d_6|1111110\rangle + d_7|1111111\rangle)] \end{aligned} \quad (4)$$

Step 1: In order to achieve remote teleportation on this three qubit gate, Alice first applies controlled-NOT gate C_{aA}^N (‘a’ as the control bit and ‘A’ as target bit) and Bob applies controlled-NOT gate C_{bB}^N . Step 2 and 3 are given in table 1 and 2 respectively.

The pictorial representation of for simultaneous $C^{\mathcal{H}}$ teleportation through parallel Bell state network has been depicted in figure 1.

The simultaneous remote teleportation of generalized controlled-Unitary gate, from two parties to one, consumes 2 ebits represented by the states $|\Phi\rangle_{AC_1}$ and $|\Phi\rangle_{BC_2}$, and total 4 cbits to communicate the measurement outcomes.

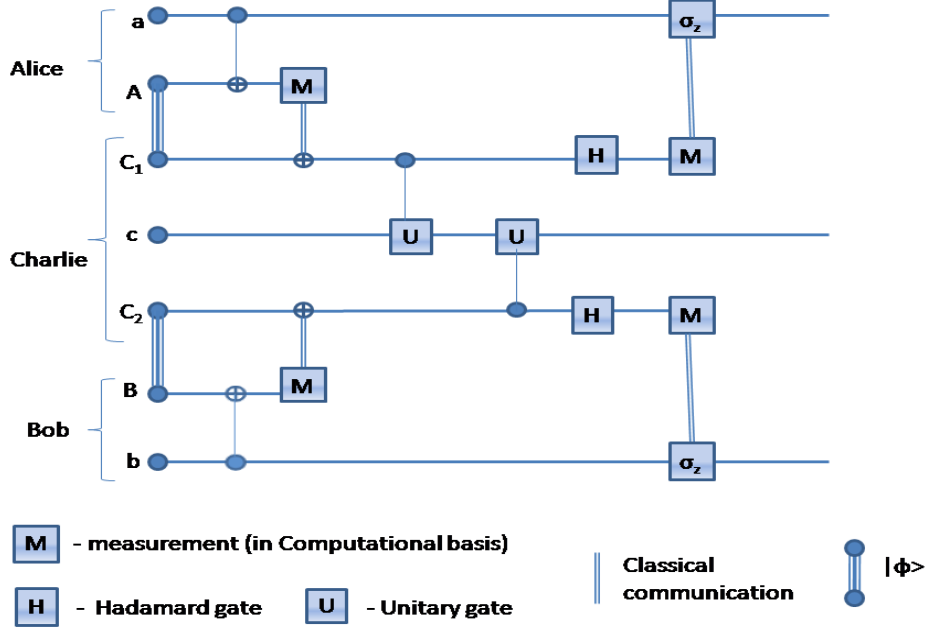


Figure 1: Simultaneous C^U teleportation through parallel network.

Table 1: The outcomes of the measurements of qubits A and B performed by Alice and Bob with the corresponding local operation applied by Charlie and the combined state obtained by them (here $\mathcal{U}|\psi\rangle = |\bar{\psi}\rangle$ and $\mathcal{U}^2|\psi\rangle = |\widetilde{\psi}\rangle$)
Step 2

Outcomes of measurements on A and B	Local operations after measurements	Combined state of qubits a,b,c,C1,C2
$ 00\rangle_{AB}$	$C_{C_1c}^U C_{C_2c}^U$	
$ 01\rangle_{AB}$	$C_{C_1c}^U C_{C_2c}^U \sigma_x^{C_2}$	$d_0 00000\rangle + d_1 00001\rangle + d_2 0101\bar{0}\rangle + d_3 0101\bar{1}\rangle$
$ 10\rangle_{AB}$	$C_{C_1c}^U C_{C_2c}^U \sigma_x^{C_1}$	$+d_4 1010\bar{0}\rangle + d_5 1010\bar{1}\rangle + d_6 1111\bar{0}\rangle + d_7 1111\bar{1}\rangle$
$ 11\rangle_{AB}$	$C_{C_1c}^U C_{C_2c}^U \sigma_x^{C_1} \sigma_x^{C_2}$	

Table 2: The outcomes of the measurements of qubits C_1 and C_2 performed by Charlie and the local operation applied by Alice and Bob to obtain the gate teleportation ($\mathcal{U}|\psi\rangle = |\tilde{\psi}\rangle$ and $\mathcal{U}^2|\psi\rangle = |\widehat{\psi}\rangle$)

Step 3

Outcomes of measurements on C_1 and C_2	Local operations after measurements	Combined state of qubits a,b,c
$ ++\rangle_{C_1 C_2}$	\mathcal{I}	
$ +-\rangle_{C_1 C_2}$	σ_Z^b	$d_0 000\rangle + d_1 001\rangle + d_2 01\bar{0}\rangle + d_3 01\bar{1}\rangle$
$ - + \rangle_{C_1 C_2}$	σ_Z^a	$+d_4 10\bar{0}\rangle + d_5 10\bar{1}\rangle + d_6 11\tilde{0}\rangle + d_7 11\tilde{1}\rangle$
$ --\rangle_{C_1 C_2}$	$\sigma_Z^b \sigma_Z^a$	

n -party generalized gate teleportation :

Here, we generalize the simultaneous teleportation protocol of $(n - 1)$ control parties and one target party, using the above parallel network. Each of the control parties share a Bell state ($|\Phi\rangle$) with the target party, the unknown state being,

$$|\chi\rangle = \sum_{m=0}^{2^n-1} a_m |i\rangle, \quad (5)$$

where ‘ i ’ is the binary representation of decimal number ‘ m ’. In this state, each of the first $(n - 1)$ qubits are possessed by the $(n - 1)$ control parties, the target party possesses the last qubit.

Each of the control parties first apply CNOT gates, taking the unknown qubit as control, then measure their Bell shared qubit in computational basis and communicate the outcomes to the target party. The target party applies σ_x on the Bell shared qubits if the measurement outcomes of the corresponding shared qubit is $|1\rangle$. After that the party applies simultaneous $C^{\mathcal{U}}$ gate $(n-1)$ times, taking the shared Bell qubit as control, and the unknown qubit as target. Then the target party does measurement of all shared qubits in Hadamard basis. After communicating the outcomes of the measurement, control parties apply σ_z gate if the outcome is $|-\rangle$ of the corresponding Bell shared qubit. The teleportation consumes $(n - 1)$ ebit and $2(n - 1)$ cbits.

3 SIMULTANEOUS TELEPORTATION OF CONTROLLED-HERMITIAN GATE USING SERIES ENTANGLEMENT NETWORK

Here, we illustrate the scheme for the simultaneous teleportation of controlled-Hermitian (as well as unitary) gates from two parties to one, where the entangled network is in series connection. Suppose, Alice, Bob and Charlie possess qubits 1,4 and 7 respectively of the arbitrary state,

$$|\psi\rangle_{147} = (d_0|000\rangle + d_1|001\rangle + d_2|010\rangle + d_3|011\rangle + d_4|100\rangle + d_5|101\rangle + d_6|110\rangle + d_7|111\rangle) \quad (6)$$

with $\sum_{i=0}^7 |d_i|^2 = 1$. Alice and Bob share a Bell state between their respective qubits 2 and 3; Bob and Charlie share a Bell state between their respective qubits 5 and 6 :

$$|\Phi\rangle_{23} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle), \quad (7)$$

and

$$|\Phi\rangle_{56} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle). \quad (8)$$

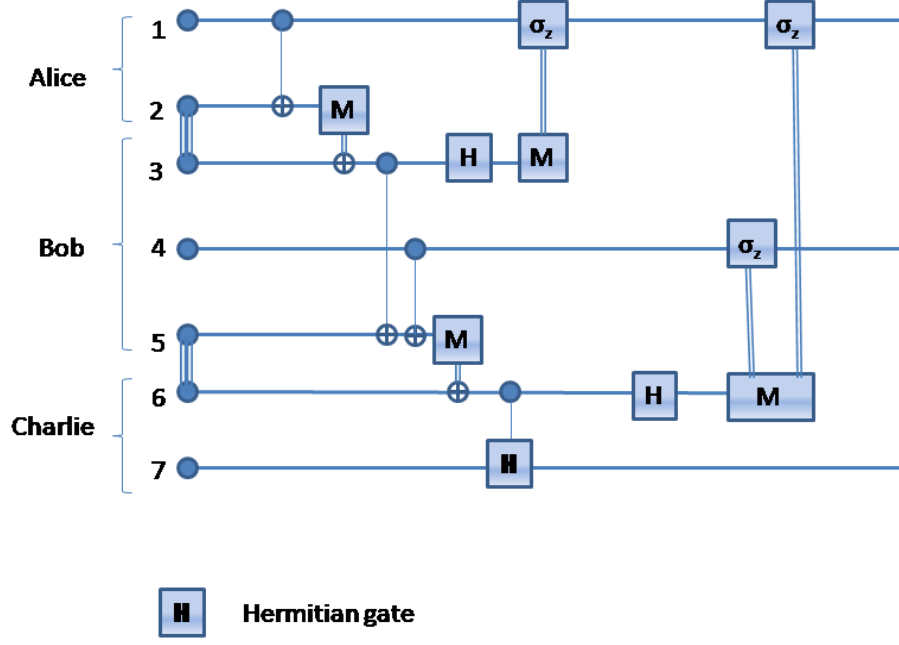


Figure 2: Simultaneous C^H teleportation through series network.

Here, Alice shares entanglement with another control party, Bob. And Bob shares entanglement with the target party, Charlie. Thus there is no direct entanglement between Alice and Charlie, which makes a series entanglement connection.

Now the combined state of all the qubits possessed by Alice, Bob and Charlie is given by,

$$\begin{aligned}
 |\zeta\rangle_{1234567} &= (|\psi\rangle_{147} \otimes |\Phi\rangle_{23} \otimes |\Phi\rangle_{56}) \\
 &= \frac{1}{2}[(d_0|0000000\rangle + d_1|0000001\rangle + d_2|0001000\rangle + d_3|0001001\rangle \\
 &\quad + d_4|1000000\rangle + d_5|1000001\rangle + d_6|1001000\rangle + d_7|1001001\rangle) \\
 &\quad + (d_0|0000110\rangle + d_1|0000111\rangle + d_2|0001110\rangle + d_3|0001111\rangle \\
 &\quad + d_4|1000110\rangle + d_5|1000111\rangle + d_6|1001110\rangle + d_7|1001111\rangle) \\
 &\quad + (d_0|0110000\rangle + d_1|0110001\rangle + d_2|0111000\rangle + d_3|0111001\rangle \\
 &\quad + d_4|1110000\rangle + d_5|1110001\rangle + d_6|1111000\rangle + d_7|1111001\rangle) \\
 &\quad + (d_0|0110110\rangle + d_1|0110111\rangle + d_2|0111110\rangle + d_3|0111111\rangle \\
 &\quad + d_4|1110110\rangle + d_5|1110111\rangle + d_6|1111110\rangle + d_7|1111111\rangle)].
 \end{aligned} \tag{9}$$

Step 1: To achieve the goal of complete remote teleportation of this three qubit gate, Alice applies controlled-NOT gate C_{12}^N on her qubits 1 and 2 and then measures qubit 2 in computational basis. Step 2, 3 and 4 are shown in table 3, 4 and 5 respectively.

The pictorial representation of local operations and measurements for simultaneous C^H teleportation through series network of Bell states has been depicted in figure 2. The simultaneous remote teleportation of controlled-Hermitian gate from two parties to one consumes 2 ebits and total 5 cbits to communicate the measurement outcomes.

n -party generalized gate teleportation :

The generalized protocol of simultaneous C^H gate teleportation using the above series network is given in figure 3, where the unknown state given in eq. 5. For n parties the communication cost is $(n - 1)$ ebits and $(n^2 + n - 2)/2$ cbits. It is worth emphasizing the interesting feature of the series network, which enables a party, not directly connected to the target, to control an operation on the same.

Table 3: The outcomes of the measurements of qubit 2, local operations by Bob and the combined state obtained by them

Step 2

Outcomes of measurements on 2	Local operations after measurements	Combined state after measurement and operations
$ 0\rangle_2$	$C_{35}^N C_{45}^N$	$d_0 000000\rangle + d_1 000001\rangle + d_2 001100\rangle + d_3 001101\rangle$ $+d_4 000110\rangle + d_1 000111\rangle + d_2 001010\rangle + d_3 001011\rangle$
$ 1\rangle_2$	$C_{35}^N C_{45}^N \sigma_x^3$	$+d_4 110100\rangle + d_5 110101\rangle + d_6 111000\rangle + d_7 111001\rangle$ $+d_4 110010\rangle + d_5 110011\rangle + d_6 111110\rangle + d_7 111111\rangle$

Table 4: The outcomes of the measurements of qubit 5 in computational basis, local unitary gates applied by Charlie and the combined state obtained by them

Step 3

Outcomes of measurements on 5	Local operations after measurements	Combined state after measurement and operations
$ 0\rangle_{AB}$	C_{67}^H	$d_0 00000\rangle + d_1 00001\rangle + d_2 0011\rangle\mathcal{H} 0\rangle + d_3 0011\rangle\mathcal{H} 1\rangle$
$ 1\rangle_{AB}$	$C_{67}^H \sigma_x^6$	$+d_4 1101\rangle\mathcal{H} 0\rangle + d_5 1101\rangle\mathcal{H} 1\rangle + d_6 11100\rangle + d_7 11101\rangle$

Table 5: The outcomes of the measurements of qubits 3 and 6 in Hadamard basis performed by Bob and Charlie and the corresponding unitary operations to obtain the desired state

Step 4

Outcomes of measurements on 3 and 6	Local operations after measurements	Combined state of qubits a,b,c after measurement and operations
$ ++\rangle_{36}$ $ +-\rangle_{36}$ $ - + \rangle_{36}$ $ -- \rangle_{36}$	\mathcal{I} $\sigma_z^1 \sigma_z^4$ σ_z^1 σ_z^4	$d_0 000\rangle + d_1 001\rangle + d_2 01\rangle\mathcal{H} 0\rangle + d_3 01\rangle\mathcal{H} 1\rangle$ $+d_4 10\rangle\mathcal{H} 0\rangle + d_5 10\rangle\mathcal{H} 1\rangle + d_6 110\rangle + d_7 111\rangle$

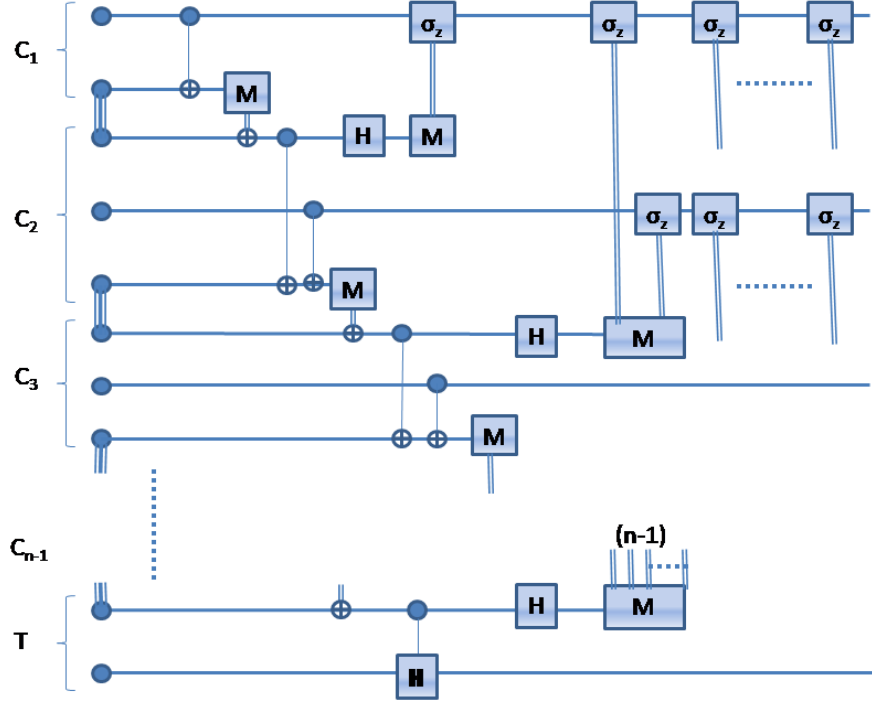


Figure 3: Generalized simultaneous C^H teleportation using the series network

We could not find any protocol where controlled-Unitary gate can be teleported using this entanglement cost. So, from the above protocol, we can say that the Unitary as well as Hermitian operators has significance in series entangled network. This operator has the additional property of involution (i.e., the operator is same as its inverse), which is responsible for making this protocol deterministic. Most of the important gates like controlled-Pauli gates, controlled-Hadamard gate etc., belong to this category, making this implementation powerful.

4 TOFFOLI GATE TELEPORTATION USING SERIES ENTANGLEMENT NETWORK

It has been shown that a more general form of Toffoli gate, i.e., controlled-controlled-Unitary gate can be deterministically teleported in parallel network, using two Bell pairs (2 ebits of entanglement) and 4 cbits to communicate the measurement outcomes [10]. In this present protocol we implement that same non-local gate using series entanglement distribution with the same communication cost, given in figure 4.

For illustration, we consider the qubit distribution described in eq. 9 :

$$|\zeta\rangle_{1234567} = (|\psi\rangle_{147} \otimes |\Phi\rangle_{23} \otimes |\Phi\rangle_{56}) \quad (10)$$

Step 1: Alice first applies controlled-NOT gates C_{12}^N and measure her qubit 2 in computational basis. Other steps are given in table 6 to 9.

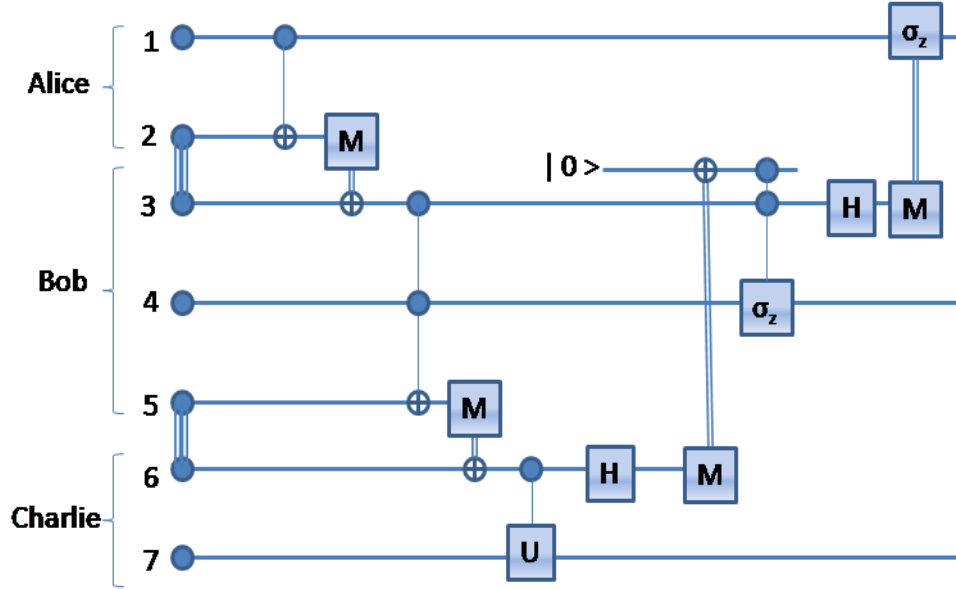


Figure 4: Controlled-controlled-Unitary gate teleportation through series network.

Table 6: The outcomes of the measurements of qubit 2 with unitary gate performed by Bob and the combined state after operations.

Step 2

Outcomes of measurements on qubit 2	Local operations by Bob	Combined state obtained after measurements and operations
$ 0\rangle_2$	C_{345}^N	$d_0 000000\rangle + d_1 000001\rangle + d_2 001000\rangle + d_3 001001\rangle$ $+ d_0 000110\rangle + d_1 000111\rangle + d_2 001110\rangle + d_3 001111\rangle$ $+ d_4 110000\rangle + d_5 110001\rangle + d_6 111100\rangle + d_7 111101\rangle$ $+ d_4 110110\rangle + d_5 110111\rangle + d_6 111010\rangle + d_7 111011\rangle$
$ 1\rangle_2$	$C_{345}^N \sigma_x^3$	

Table 7: The outcomes of the measurements of qubit 5 with unitary gate performed by Charlie and the combined state after operations.

Step 3

Outcomes of measurements on qubit 5	Local operations by Bob	Combined state obtained after measurements and operations
$ 0\rangle_5$	$C_{67}^{\mathcal{U}}$	$d_0 00000\rangle + d_1 00001\rangle + d_2 00100\rangle + d_3 00101\rangle + d_4 11000\rangle + d_5 11001\rangle + d_6 11110\rangle + d_7 11111\rangle$
$ 1\rangle_5$	$C_{67}^{\mathcal{U}}\sigma_x^6$	

Table 8: The outcomes of the measurements of qubit 6 in Hadamard basis with unitary gate performed by Bob and the combined state after operations.

Step 4

Outcomes of measurements on qubit 6	Local operations by Bob	Combined state obtained after measurements and operations
$ +\rangle_6$	\mathcal{I}	$d_0 0000\rangle + d_1 0001\rangle + d_2 0010\rangle + d_3 0011\rangle + d_4 1100\rangle + d_5 1101\rangle + d_6 1110\rangle + d_7 1111\rangle$
$ -\rangle_6$	$C_{34}^{\sigma_z}$	

Table 9: The outcomes of the measurements of qubit 3 in Hadamard basis with unitary gate performed by Alice and the combined desired state obtained by the three parties

Step 5

Outcomes of measurements on qubit 3	Local operations by Bob	Combined state obtained after measurements and operations
$ +\rangle_3$	\mathcal{I}	$d_0 000\rangle + d_1 001\rangle + d_2 010\rangle + d_3 011\rangle + d_4 100\rangle + d_5 101\rangle + d_6 110\rangle + d_7 111\rangle$
$ -\rangle_3$	σ_z^1	

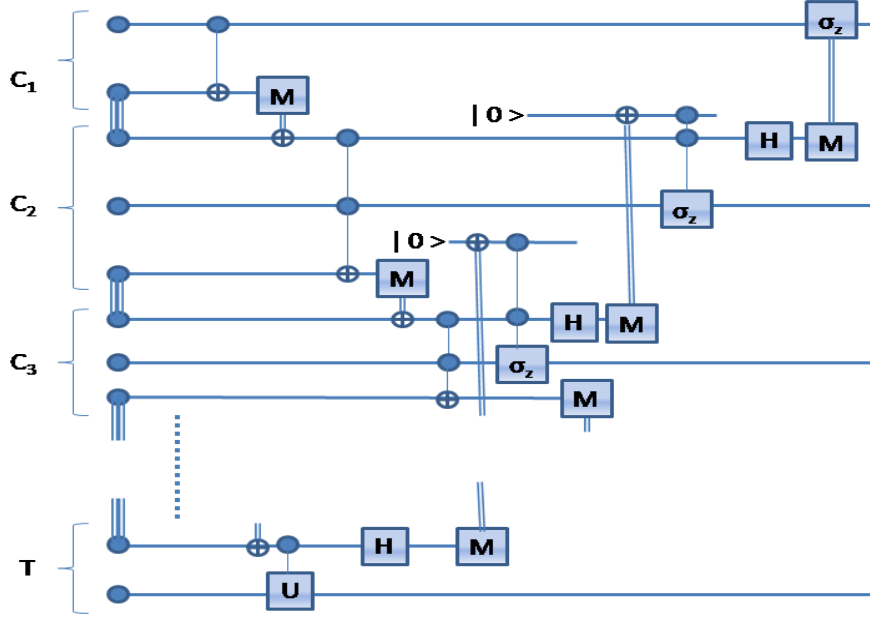


Figure 5: Generalized controlled-Unitary gate teleportation using series entangled network.

***n*-party generalized gate teleportation :**

We now generalize the above procedure to teleport a ‘*n*’-qubit gate, where $(n - 1)$ qubits are controls and the unitary operator acts on the target qubit, only if, all the control qubits are $|1\rangle$ s. The protocol is shown in figure 5 and for *n*-qubit gate the communication cost is $(n - 1)$ ebits and $2(n - 1)$ cbits which is same as for the parallel network, given in Ref.[10].

5 CONCLUSION

In conclusion, we have demonstrated gate teleportation protocols of simultaneous controlled gate operations in two different configurations, which are efficient, as evident from the associated communication and entanglement costs. All the protocols have been generalized to ‘*n*’-qubit gates. Interestingly, in the series configuration, the implemented generalized gate and communication cost is different from the parallel distribution for the simultaneous gate teleportation. Optimal protocol can be investigated for the simultaneous controlled-Unitary gate teleportation in series entangled channels. In parallel to the qubit case of remote state preparation, the gate teleportation with partial information needs to be investigated [7,8]. The fact that, Bell states are realized in laboratory conditions makes our protocol experimentally achievable [3]. In future, we would like to study the teleportation protocols with minimum communication costs of other important gates, which do not belong to this present category. The question of teleportation witness [22,23,24] for the present protocol needs careful study.

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7 REFERENCES:

- [1] M. A. Nielsen and I. L. Chuang, ‘Quantum computation and quantum information’, (Cambridge University Press, 2002).
- [2] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, ‘Teleporting an unknown quantum state via dual classical and Einstein-Podolsky-Rosen channels’, *Phys. Rev. Lett.* 70, 1895 (1993).
- [3] D. Bouwmeester, J. W. Pan, K. Mattle, M. Eibl, H. Weinfurter, and A. Zeilinger, ‘Experimental quantum teleportation’, *Nature (London)* 390, 575 (1997).
- [4] C. H. Bennett, and S. J. Wiesner, ‘Communication via one and two particle operators on Einstein-Podolsky-Rosen states’, *Phys. Rev. Lett.* 69, 2881 (1992).
- [5] S. Bandyopadhyay, ‘Teleportation and secret sharing with pure entangled states’, *Phys. Rev. A* 62, 012308 (2000).
- [6] M. Hillery, V. Bužek, and A. Berthiaume, ‘Quantum secret sharing’, *Phys. Rev. A* 59, 1829 (1999).
- [7] A. K. Pati, ‘Minimum classical bit for remote preparation and measurement of a qubit’, *Phys. Rev. A* 63, 014302 (2000).
- [8] C. H. Bennett, P. Hayden, D. W. Leung, P. W. Shor, A. Winter, ‘Remote preparation of quantum states’, *Information Theory, IEEE Transactions on*, *IEEE* 51, 56 (2005); P. K. Panigrahi, S. Karumanchi, S. Muralidharan, ‘Minimal classical communication and measurement complexity for quantum information splitting of a two-qubit state’, *Pramana* 73, 499 (2009).
- [9] A. Barenco, C. H. Bennett, R. Cleve, D. P. DiVincenzo, Norman Margolus, Peter Shor, Tycho Sleator, John A. Smolin, and Harald Weinfurter, ‘Elementary gates for quantum computation’, *Phys. Rev. A* 52, 3457 (1995).
- [10] J. Eisert, K. Jacobs, P. Papadopoulos and M. B. Plenio, ‘Optimal local implementation of nonlocal quantum gates’, *Phys. Rev. A* 62, 052317 (2000).
- [11] D. Collins, N. Linden, and S. Popescu, ‘Nonlocal content of quantum operations’, *Phys. Rev. A* 64, 032302 (2001); J. Eisert et al., *Phys. Rev. A* 62, 052317 (2000).
- [12] Ming-Yong Ye, Yong-Sheng Zhang, and Guang-Can Guo, ‘Efficient implementation of controlled rotations by using entanglement’, *Phys. Rev. A* 73, 032337 (2006).
- [13] W. Dür and J. I. Cirac, ‘Nonlocal operations: Purification, storage, compression, tomography, and probabilistic implementation’, *Phys. Rev. A* 64, 012317 (2001).
- [14] A. Chefles, C. R. Gilson, and S. M. Barnett, ‘Entanglement, information, and multiparticle quantum operations’, *Phys. Rev. A* 63, 032314 (2001).
- [15] J. I. Cirac, W. Dür, B. Kraus, and M. Lewenstein, ‘Entangling Operations and Their Implementation Using a Small Amount of Entanglement’, *Phys. Rev. Lett.* 86, 3 (2001).
- [16] Chui-Ping Yang, ‘A new protocol for constructing nonlocal n-qubit controlled-U gates’, *Phys. Lett. A* 372, 1380 (2008).
- [17] D. W. Berry, ‘Implementation of multipartite unitary operations with limited resources’, *Phys. Rev. A* 75, 032349 (2007).
- [18] G.-F. Dang and H. Fan, ‘Remote controlled-NOT gate of d-dimension’, e-print arXiv:0711.3714v2 (2008).
- [19] An Min Wang, ‘Combined and controlled remote implementations of partially unknown quantum operations of multiqubits using Greenberger-Horne-Zeilinger states’, *Phys. Rev. A* 75, 062323 (2007).
- [20] Li Yu, R. B. Griffiths and S. M. Cohen, ‘Efficient implementation of bipartite nonlocal unitary gates using prior entanglement and classical communication’, *Phys. Rev. A* 81, 062615 (2010).
- [21] Xinlan Zhou, Debbie W. Leung, and Isaac L. Chuang, ‘Methodology for quantum logic gate construction’, *Phys. Rev. A* 62, 052316 (2000).
- [22] Chui-Ping Yang, ‘A new protocol for constructing nonlocal n-qubit controlled-U gates’, *Phys. Lett. A* 372, 1380 (2008).
- [23] Lin Chen and Yi-Xin Chen, ‘Probabilistic implementation of a nonlocal operation using a nonmaximally entangled state’, *Phys. Rev. A* 71, 054302 (2005).
- [24] Berry Groisman and Benni Reznik, ‘Implementing nonlocal gates with nonmaximally entangled states’, *Phys. Rev. A* 71, 032322 (2005).
- [25] Chun-mei Yao, and Bin-fang Cao, ‘Efficient implementation of a nonlocal gate with nonmaximal entanglement’, *Phys. Lett. A* 373, 1011 (2009).
- [26] J. Jang, J. Lee, M. S. Kim, and Y.-J. Park, ‘Implementation of non-local operations for arbitrary high-dimensional systems with qubit quantum channel’, e-print, arXiv:quant-ph/0101107v1 (2001).
- [27] Yun-Feng Huang, Xi-Feng Ren, Yong-Sheng Zhang, Lu-Ming Duan, and Guang-Can Guo, ‘Experimental Teleportation of a Quantum Controlled-NOT Gate’, *Phys. Rev. Lett.* 93, 240501 (2004).

- [28] N. Ganguly, S. Adhikari, A. S. Majumdar and J. Chatterjee, ‘Entanglement witness operator for quantum teleportation’, Phys. Rev. Lett. 107, 270501 (2011).
- [29] A. Kumar, S. Adhikari and P. Agrawal, ‘Generalized form of optimal teleportation witness’, arXiv:1204.0983v2 [quant-ph] (2012).
- [30] M. Bourennane, M. Eibl, C. Kurtsiefer, S. Gaertner, H. Weinfurter, O. Gühne, P. Hyllus, D. Brúß, M. Lewenstein, and A. Sanpera, ‘Experimental Detection of Multipartite Entanglement using Witness Operators’, Phys. Rev. Lett. 92, 8 (2004).